D. Phil. Course Work in Mathematics
(in partial fulfillment for the award of the degree of D. Phil.)
One Semester Course

A candidate will be required to offer

(1) two out of three core course papers MAT701C, MAT702C and MAT703C.
(2) one optional paper.
(3) a course work on "Research Methodology".

6 credits for each one of the Papers MAT701C, MAT702C and MAT703C: Two lectures and one tutorial per week; 100 marks for each paper; A total of 40 periods for a paper

6 credits for an optional paper: Two lectures and one tutorial per week; 100 marks

Research Methodology and Communication skill and seminars: 200 marks
8 credits "Research Methodology**"**
4 credits "Communication skill and seminars"

Grand total: 500 marks

Examination: 40 marks internal assessment as per

Duration of Examination paper: Three Hours

Course Structure
Course Code/Name of the Course/Credits (L-T-P-C)

MAT701C	**Algebra**	**2-1-0-6**
MAT702C	**Analysis**	**2-1-0-6**
MAT703C	**Topology and Geometry**	**2-1-0-6**
MAT704O	**Complete Invariance Property**	**2-1-0-6**
MAT705O	**Fixed Point Theory**	**2-1-0-6**
MAT706O	**Viscous Flow in Porous Media and MHD**	**2-1-0-6**
MAT707O	**Dynamical Systems**	**2-1-0-6**
MAT708O	**Group Theory**	**2-1-0-6**
MAT709O	**Structures on Manifolds**	**2-1-0-6**
MAT710O	**Foundations of Finsler Geometry**	**2-1-0-6**
MAT711C	**RESEARCH METHODOLOGY**	**4-2-0-12**

DETAILED SYLLABUS

MAT701C: ALGEBRA

Review of basic concepts in algebra and linear algebra; Importance, significance and applications of Lagrange's Theorem, Fundamental Theorem of Homomorphism, Isomorphism Theorems, Cayley's Theorem, Sylow Theorems, Maximal and prime ideals, Principal ideal domains, Unique factorization domains, Euclidean domain, field extensions, finite fields, Galois theory.

Cayley-Hamilton Theorem, Sylvester's law of inertia.

MAT702C: ANALYSIS

Review of basic concepts in analysis including real, complex and functional: Importance, significance and applications of Mean Value Theorem, Weierstrass Approximation Theorem, Inverse Function Theorem, Implicit Function Theorem, Fatou's Lemma, Monotone Convergence Theorem, Lebesgue Convergence Theorem, Fubini's Theorem, Cauchy-Riemann Equations, Cauchy Formula, Morera's Theorem, Laurent's Series, Maximum Modulus Theorem, Hahn-Banach Theorem, Open Mapping Theorem, Closed Graph Theorem, Uniform Boundedness Principle, Riesz Representation Theorem (with sketch of proofs).

Deviations of infinite dimensional Hilbert spaces from that of finite dimensional Hilbert spaces.

MAT703C: TOPOLOGY AND GEOMETRY

Review of General Topology: Basic notions including subspace, product and quotient, Attaching spaces, Spheres, reduced suspension, real and complex projective spaces and generalized torus as attaching spaces, finite CW-complexes and cellular maps.

Statements of Urysohn's Lemma, Tietze's Extension Theorem, Embedding Lemma, Urysohn Metrization Theorem, Stone-Čech compactification, their significance.

Classical matrix groups, preliminaries in a topological group, Exponential of a matrix, Lie Groups and their actions on smooth manifolds, Cylinder, Torus, Mobius band and Klein bottle as orbit spaces of a properly discontinuous group action of discrete groups on manifolds, one-parameter subgroups of a Lie group, one-parameter groups acting on a manifold, ODE's and manifolds (with sketch of proofs).

MAT704O: COMPLETE INVARIANCE PROPERTY

Homotopy of maps, retraction, deformation retraction, strong deformation retraction, examples and elementary results.

Brouwer's fixed point theorem on a disc, Fixed point set of a self-continuous map on a disc, complete invariance property (CIP), complete invariance property w.r.t. a homeomorphism (CIPH), Examples and counter-examples (Hilbert cube, Cantor set, one-dimensional Peano spaces).

Preservation of these properties by subspaces and products, compactification and CIP, Ward's Lemma.

Uniform flow, compact metrizable groups and CIPH.

References:

1. L. E. Ward Jr., Fixed-point sets, Pacific Math. Jour. 47(1973), 553-565.
2. J. R. Martin, Fixed point sets of homeomorphisms of metric products, Proc. Amer. Math. Soc. 103(1988), 1293-1298.
3. Alex Chigogidze, Karl H. Hofmann and John R. Martin, Compact groups and fixed point sets, Trans. Amer. Math. Soc. 349(1997), 4537-4554.
4. Alex Chigogidze and John R. Martin, Fixed point sets of autohomeomorphisms of uncountable products, Topology and Appl. 80(1997), 63-71.

MAT705O: FIXED POINT THEORY

Review of basic concepts in Functional Analysis:
Compactness in metric spaces, measures of non-compactness, Normed spaces, Banach spaces, Uniformly convex and reflexive Banach spaces, Examples, Contraction and nonexpansive mappings, Condensing maps, Set-Valued mappings, Examples.

Significance and applications of Banach's contraction principle, Browder-Gohde-Kirk's theorem, The demiclosedness principle, Schauder's fixed point theorem, Sadovskii's fixed point theorem, Nadler's contraction principle for set valued mappings, Caristi-Ekeland principle, Eberlein-Shmulyan Theorem, Mann and Ishikawa iteration procedures, Bruck's theorem, Darbo's theorem, Kirzbraun-Valentine theorem, Lim's theorem, Menger's theorem, Nonlinear Mean Ergodic theorem, Kadec-Klee norm, Zizler's Theorem.

References:

1. K. Goebel and W. A. Kirk, Topics in Metric Fixed Point Theory, Cambridge Studies in Advances Mathematics, No. 28, 1990.
2. A. Granas, Fixed Point Theory, Springer Monographs in Mathematics, 2003.
3. V. Berinde, Iterative Approximation of Fixed points, Lecture Notes in Mathematics, No. 1912, Springer, 2007.

MAT706O: VISCOUS FLOW IN POROUS MEDIA AND MHD

Review of basic concepts in fluid mechanics, Principles of conservations of mass, momentum (linear and angular) and energy and their significance, Stress and Strain Tensors, Newtonian and non-Newtonian fluids, Constitutive equations, Navier-Stokes equations of motion, Boundary conditions in fluid mechanics, Concept of dynamical similarity and dimensionless numbers and their significance in the fluid dynamics, Some exact solutions of Navier-Stokes equations, Stokes flow, Boundary layer concept.

Mechanics of fluid flow through porous medium: Porosity and permeability, seepage velocity, effective viscosity, Darcy's law, Brinkman's equation, Darcy Forchhiemer's equation, Darcy-Lapwood-Forchhiemer-Brinkman's equation, Boussinesq approximation, Convection, filtration.

Introduction to Magnetohydrodynamics (MHD), MHD parameters and their significance.

References:

1. J. Happel and H. Brenner, Low Reynolds Number Hydrodynamics, Kluwer Academic Publishers Group, Dordrecht, The Netherlands, 1983
2. Z. U. A. Warsi, Fluid Dynamics, CRC Press, 2005.
3. D. A. Nield and A. Bejan, Convection in Porous Media, Springer-Verlag, 1992.
4. S. Chandrasekhar, Hydrodynamics and Magnetohydrodynamics Stability, Dover Publications, 1981.

MAT707O: DYNAMICAL SYSTEMS

Definition of a dynamical system, Iterates of a function, recursion equation, phase portraits, the logistic function, review of the topology of the real line and analysis of real functions, fixed (or equilibrium) points, periodic points, asymptotic points, stable sets, graphical analysis.

Sarkovskii's theorem, sufficient condition for a function on a closed interval to have a unique fixed point, dynamical information from a differentiable function, hyperbolic periodic points, attracting periodic points.

Review of basic concepts of general measure theory, extension of a measure on semialgebra to the generated algebra, measure preserving transformations, definitions and examples, construction of a new measure preserving transformation from given ones, recurrence property of a measure preserving transformation, Poincaré's recurrence theorem, introduction to ergodicity.

Books Recommended:

1. Richard A. Holmgren, A First Course in Discrete Dynamical Systems, Springer-Verlag, 1994.
2. H. L. Royden and P. M. Fitzpatrick, Real Analysis, 4th Ed., Prentice Hall, 2010.
3. Peter Walters, An Introduction to Ergodic Theory, Springer, 1982.

Further Reading:

1. R. L. Devaney, An Introduction to Chaotic Dynamical Systems, 2nd Ed., Addison Wesley, 1989.
2. K. R. Parthasarthy, Introduction to Probability and Measure, Hindustan Book Agency, 2005.

MAT708O: GROUP THEORY

Groups defined by generators and relations (some examples), symmetric and alternating groups and their automorphisms groups, free product with amalgamations, finite p-groups, central product of groups, extra-special p-groups, special p-groups, Sharply 2-transitive groups.

Group extensions, Cohomology of groups and some of its applications.

References:

1. Michio Suzuki, Group Theory, Vol. I, Springer, New York, 1980.
2. Michio Suzuki, Group Theory, Vol. II, Springer, New York, 1986.
3. D. J. S. Robinson, A Course in the Theory of Groups, Graduate Text in Mathematics, No. 80, Springer, New York, 1996.
4. K. W. Grenberg, Cohomological Topics in Group Theory, Lectures Notes in Mathematics, No. 143, Springer, Berlin 1970.

MAT709O: STRUCTURES ON MANIFOLDS

Complex Manifolds:

Almost complex manifolds, Nijenhuis tensor, eigen values of an almost complex structure, existence theorem and integrability condition of an almost complex structure, complex manifolds, almost Hermitian manifolds, some well known classes of almost Hermitian manifolds and their curvature properties, (almost Kaehler manifolds, Kaehler manifolds, nearly Kaehler manifolds), para-Kaehler manifolds.

Contact Manifolds:

Contact metric manifolds, almost contact manifolds, torsion tensor of an almost contact metric manifold, killing vector field, properties of structure tensor ϕ, curvature properties of contact metric manifolds, para-contact manifolds.

K-Contact Manifolds:

Characterizations of K-contact manifolds, curvature properties, sectional curvature, locally symmetric and semi-symmetric K-contact manifolds.

Sasakian Manifolds:

Curvature properties, ϕ-sectional curvature of a Sasakian manifold, semi-symmetric and Weyl semi-symmetric Sasakian manifolds, C-Bochner curvature tensor on a Sasakian manifold.

Submanifolds of Kaehler and Sasakian Manifolds:

Submanifolds of Riemannian manifolds, equations of Gauss, Codazzi and Ricci, some well known classes of submanifolds of Kaehler and Sasakian manifolds (invariant, anti-invariant and CR-submanifolds).

References:

1. K. Yano and M. Kon, Structures on Manifolds, World Scientific, 1984.
2. D. E. Blair, Riemannian Geometry of Contact and Symplectic Manifolds, Progress in Mathematics, Vol. 203, Birkhäuser Inc., Boston, MA, 2002.
3. R. S. Mishra, Structures on Differentiable Manifolds and their Applications, Chandrama Prakashan, Allahabad, 1984.
4. U. C. De and A. A. Shaikh, Complex Manifolds and Contact Manifolds, Narosa Publishing House, New Delhi, 2009.

MAT710O: FOUNDATIONS OF FINSLER GEOMETRY

UNIT ONE: Differentiable manifolds, Lie transformation groups, Fiber bundles, Linear connections, Structure equations.

UNIT TWO: Finsler bundles, Distributions in the Finsler bundles, Various connections relating to the Finsler bundles, Finsler connections, Torsion and curvatures of Finsler connections, Bianchi identities of Finsler connections.

UNIT THREE: Homogeneity of Finsler connections, The characteristic conditions for Finsler connections, Difference of two Finsler connections, Finsler metrics, The Cartan connection, The Rund and Berwald connections.

UNIT FOUR: The *N*-decompositions of tensors, Linear connections of Finsler type, Lifts of Finsler metrics, Isotropy of Finsler spaces.

UNIT FIVE: Riemannian Spaces and locally Minkowski spaces, Berwald spaces and Landsberg spaces, Finsler spaces of scalar curvature, Intrinsic fields of orthonormal frame, Two-dimensional Finsler spaces, C-reducibility of Finsler spaces.

Books Recommended:

1. M. Matsumoto, Foundations of Finsler Geometry and special Finsler Spaces, Kaiseisha Press, Otsu, 1986.
2. P. L. Antonelli (Ed.), Handbook of Finsler Geometry, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2003.
3. D. Bao, S. S. Chern and Z. Shen, An Introduction to Riemann-Finsler Geometry, Springer-Verlag, New York, 2000.
4. H. Rund, The Differential Geometry of Finsler Spaces, Springer-Verlag, Berlin, 1959.

MAT711C: RESEARCH METHODOLOGY

2 credits for each of the following four Sections

200 marks; 50 for each Section

To be directed and assessed by the DPC

One lecture plus one class work (presentation) per week for each of the four sections

1. Review of Literature: Searching related research papers, survey articles, scope for future work with historical notes.
2. Being acquainted with review of published articles and books in the field of research work undertaken (Mathematical Reviews, Zentralblatt, etc.), Impact factor of a journal.
3. Familiarity with certain relevant software(s) in the field of work.
4. Familiarity with Latex: Preparing a survey article for presentation; correcting galley proofs, writing comments and preparing summary and abstract of a manuscript.

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF ALLAHABAD

K.K. Azad
The Head

April 19, 2012

MINUTES OF THE MEETING OF THE BOARD OF STUDIES IN MATHEMATICS (BOS) HELD ON APRIL 19, 2012 AT 11:00 a.m. IN THE CHAMBER OF THE HEAD OF THE DEPARTMENT

Members Present:

1. Professor B. Rai
2. Professor D.P. Choudhury
3. Professor P.N. Pandey
4. Professor Neeta Singh
5. Dr. P.K. Singh
6. Dr. Mona Khare
7. Dr. R.P. Shukla
8. Dr. S.S. Shukla
9. Dr. Satya Deo
10. Dr. B.K. Sharma
11. Professor Anoop Chaturvedi
12. Professor K.K. Azad (Chairman)

MINUTES:

Under Item No. 1: The minutes of the last meeting of the Board of Studies in Mathematics held on October 21, 2011 were confirmed.

Under Item No. 2: It was resolved that the "PG SEMESTER COURSE STRUCTURE IN MATHEMATICS ALONG WITH DETAILED SYLLABUS ATTACHED AS ANNEXURE I EFFECTIVE FROM 2012-13" be adopted.

Under Item No. 3: (a) It was resolved that the "D.Phil. COURSE WORK IN MATHEMATICS IN PARTIAL FULFILLMENT FOR THE AWARD OF THE D.Phil. DEGREE ALONG WITH DETAILED SYLLABUS ATTACHED AS ANNEXURE II EFFECTIVE FROM 2012-13" be adopted.

(b) D.Phil. Examiners (Annexure III) for the following research scholars:

1. Km. Julee Srivastava (Enrolled on July 09, 2008)
2. Sri Suresh Kumar Shukla (Enrolled on July 09, 2008)
3. Km. Shivalika Saxena (Enrolled on April 09, 2009)

were recommended for appointment.

The Chairman and other members of the BOS welcomed newly appointed cognate member Professor Anoop Chaturvedi, Head of the Statistics Department, University of Allahabad.

Further, a specific mention was made of the telephonic conversations and the e- mail communications that different members had from time to time with the external members of the BOS, Professor Peeyush Chandra and Professor A.K. Srivastava, in connection with the preparation of the PG syllabus discussed and finalized today. The Board of Studies in Mathematics expressed its sense of thankfulness to them as well as to the cognate member Professor Anoop Chaturvedi for their valuable suggestions.

The meeting ended with a vote of thanks to the Chair.

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF ALLAHABAD

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ANNEXURE 1

PG SEMESTER COURSE STRUCTURE IN MATHEMATICS ALONG WITH DETAILED SYLLABUS

1. There shall be four semesters in the two-year M.A./M.Sc. course in Mathematics.
2. There will be five papers in each semester with two viva-voce examinations, one in Semester 2 and the other in Semester 4.
3. Each paper will be of 100 marks. This will include an internal test of 20 marks and 10 marks for attendance. Duration for examination of a paper will be 3 hours.
4. A total of thirty five lectures plus five periods of the interactive form are to be devoted to each paper. Four lectures per week are to be allotted to each paper.
5. A viva-voce Examination of 50 marks will be held each year, i.e, during Semester 2 and Semester 4. The Board of Examiners will consist of one External and two internal examiners recommended for appointment by the BOS. The Chairman of the Board will be the senior-most from amongst the internal examiners.
6. There shall be 500 marks each for Semester 1 and Semester 3, while 550 marks each for Semester 2 and Semester 4. Thus for the entire course it comes out to be a total of 2100 marks.
7. Five papers for Semester 1 are:
    1. MAT401C Algebra I
    2. MAT403C Complex Analysis
    3. MAT405C Point-Set Topology
    4. MAT407C Differential Geometry I
    5. MAT409C Classical Mechanics

8. Six papers for Semester 2 are:
    1. MAT402C Algebra II
    2. MAT404C Measure and Integration
    3. MAT406C Partial Differential Equations and Integral Equations
    4. MAT408C Mathematical Methods
    5. MAT410OA Tensors and Riemannian Geometry/MAT410OB Differential Topology
    6. MAT412C Viva-Voce
9. Five papers for Semester 3 are:
    1. MAT501C Advanced Linear Algebra
    2. MAT503C Functional Analysis
    3. MAT505C Theory of Ordinary Differential Equations
    4. MAT507C Differential Geometry II
    5. MAT509C Fluid Mechanics I
10. Six papers for Semester 4 are:
    1. MAT502C Wavelets
    2. MAT504OA Fluid Mechanics II/MAT504OB Algebraic Topology
    3. MAT506O/508O Optional
    4. MAT506O/508O Optional
    5. MAT510C Dissertation (to be assigned in the beginning of Semester 4)
    6. MAT512C Viva-Voce

Course Outline

1st Semester July to December

Papers	Duration (L-T-P-C)	Marks
MAT401C Algebra I	4 – 0 – 0 – 4	100
MAT403C Complex Analysis	4 – 0 – 0 – 4	100
MAT405C Point-Set Topology	4 – 0 – 0 – 4	100
MAT407C Differential Geometry I	4 – 0 – 0 – 4	100
MAT409C Classical Mechanics	4 – 0 – 0 – 4	100

2nd Semester January to May

Papers	Duration (L-T-P-C)	Marks
MAT402C Algebra II	4 – 0 – 0 – 4	100
MAT404C Measure and Integration	4 – 0 – 0 – 4	100
MAT406C Partial Differential Equations and Integral Equations	4 – 0 – 0 – 4	100
MAT408C Mathematical Methods	4 – 0 – 0 – 4	100
MAT410OA Tensors and Riemannian Geometry/ MAT4106OB Differential Topology	4 – 0 – 0 – 4	100
MAT412C Viva-Voce	0 – 0 – 0 – 2	50

3rd Semester July to December

Papers	Duration (L-T-P-C)	Marks
MAT501C Advanced Linear Algebra	4 – 0 – 0 – 4	100
MAT503C Functional Analysis	4 – 0 – 0 – 4	100
MAT505C Theory of Ordinary Differential Equations	4 – 0 – 0 – 4	100
MAT507C Differential Geometry II	4 – 0 – 0 – 4	100
MAT509C Fluid Mechanics I	4 – 0 – 0 – 4	100

4th Semester January to May

Papers	Duration (L-T-P-C)	Marks
MAT502C Wavelets	3 – 0 –1 – 4	100
MAT504OA Fluid Mechanics II/ MAT504OB Algebraic Topology	4 – 0 – 0 – 4	100
MAT506O/508O Optional	4 – 0 – 0 – 4	100
MAT506O/508O Optional	4 – 0 – 0 – 4	100
MAT510C Dissertation (to assigned in the beginning of Semester 4)	4 – 0 – 0 – 4	100
MAT512C Viva-Voce	0 – 0 – 0 – 2	50

L: Lectures per week, T: Tutorials per week, P: Practicals per week, C: Credits
List of Optional Papers:

1. Representation Theory of Finite Groups
2. Lie Algebra
3. Coding Theory
4. Algebraic Number Theory
5. Nonlinear analysis
6. Finsler Geometry

Note: The Departmental Committee shall assign a topic for dissertation along with an advisor to a Candidate in the beginning of the fourth semester. The dissertation will be evaluated by the advisor and an examiner suggested by the advisor duly approved by the BOS.